Differential Geometry 2023 - Assignment

Answer any five out of eight. Each carries 6 marks

1. Find the length of the parametrized curve $\alpha : [0, 2\pi] \to \mathbb{R}^4$ given by $(cos\ t, sin\ t, cos\ t, sin\ t)$.
2. Show that if C is connected oriented plain curve and $\tilde{C}$ is the same curve with the opposite orientation, then $l(C) = l(\tilde{C})$.
3. Compute the integral $\int_C (-x_2 dx_1 + x_1 dx_2)$ where C is the ellipse $(x_1^2/a^2) + (x_2^2/b^2) = 1$ oriented by its inward normal.
4. Let $C = f^{-1}(c)$ be a compact plane curve oriented by $\nabla f / \|\nabla f\|$, let X be the unit vector field on $U = domain(f)$ obtained by rotating $\nabla f / \|\nabla f\|$ through an angle $-\pi/2$, and let w_X be the 1-form on U dual to X. Show that $\int_C w_X = l(C)$.
5. Prove that Gauss-Kronecker curvature of an n-surface is independent of the choice of orientation if n is even.
6. Find the Gaussian curvature of $x_1^2 + x_2^2 - x_3^2 = 0$, $x_3 > 0$.
7. Find the Gaussian curvature of a cylinder over a plane curve.
8. Let $S = f^{-1}(c)$ be an oriented $n-$surface, oriented by $N = \nabla f / \|\nabla\|$. Show that $H(p) = -(1/n) div\ N$.