Differential Geometry 2023 - Minor 3

Each question carries a maximum of 5 marks. Maximum Marks 25

1. (a) Define Covariant derivative of a vector field tangent to the surface along a parametrized curve.
   (b) Define Levi-Civita parallel.
   (c) Define Covariant derivative of the tangent vector field X with respect to $v \in S_p$
   (d) Prove or disprove: In an n-plane parallel transport is path dependent.
2. Compare the curvatures of the surfaces $C_1 = x^2 + y^2 + 2(x + y) = 0$ and $C_2 = x^2 + y^2 - (x + y) = 0$, both oriented by inward normal.
3. Define parallel transport. Explain this map is well defined.
4. (a) Define the derivative ∇_v of a smooth function. Show that the map $v \mapsto \nabla_v f$ is linear.
   (b) Define the Weingarten map. Explain that this map is well defined.
5. (a) Let S^2 be the unit 2-sphere. Give parameterized curves from north pole $p = (0, 0, 1)$ to the south pole $q = (0, 0, -1)$ passing through the points $(1, 0, 0)$ and $(0, 1, 0)$.
   (b) Show that the parallel transport preserves inner product.
6. A smooth tangent vector field Xon and n-surface S is said to be a geodesic vector field if all integral curves of X are geodesics of S. Show that a smooth tangent vector field X on S is a geodesic field if and only if $D_{X(p)}X = 0$ for all $p \in S$.