Functions of Several Variable and Differential Geometry 2025 - Minor 1

Part A: *Each question carries 1 mark.*

1. Define partial derivative and directional derivative
2. Define contraction and state contraction principle.
3. State inverse function theorem.
4. State implicit function theorem.

Part B: *Each question carries 3 marks.*

5. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $f(x, y) = (x, 2x, 3x)$. Is f differentiable? If not, why? If yes, find $f'(x)$ for some $x \in \mathbb{R}^2$.
6. Suppose that f is a differentiable real function in an open set $E \subset \mathbb{R}^n$, and that f has a local maximum at a point $x \in E$. Prove that $f'(x) = 0$.

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Part C: *Each question carries 3 marks.*

7. Let $E \subset \mathbb{R}^n$ be open. Let $f : E \to \mathbb{R}^m$ is differentiable at $x \in E$. Find the matrix corresponding to $f'(x)$.

8. If f is $\mathcal{C}'$-mapping of an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^n$ and if $f'(x)$ is invertible for every $x \in E$, then $f(W)$ is open subset of $\mathbb{R}^n$ for every open set $W \subset E$.

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