Functions of Several Variable and Differential Geometry 2024 - Minor 4

Maximum Marks : 16

Part A:

1. (a) If $\alpha : I \to \mathbb{R}^{n+1}$ is a unit speed parametrized curve then the length of α is the length of the interval I. (1 mark)

   (b) Let $\alpha : (-2\pi, 2\pi) \to \mathbb{R}^2$ be defined as $\alpha(t) = (\cos t, \sin t)$. Find the length of α. (2 marks)

2. If $f : U \to \mathbb{R}$ where $U \subseteq \mathbb{R}^{n+1}$ be an open subset, and $g : \mathbb{R} \to \mathbb{R}$ are two smooth functions prove that $d(h \circ f) = (h' \circ f)df$. (2 marks)

3. Define exact 1-form and prove that the integral over of an exact 1-form over a compact connected oriented plane curve is always zero. (3 marks)

Functions of Several Variable and Differential Geometry 2024 - Minor 4

Maximum Marks : 16

Part A:

1. (a) If $\alpha : I \to \mathbb{R}^{n+1}$ is a unit speed parametrized curve then the length of α is the length of the interval I. (1 mark)

   (b) Let $\alpha : (-2\pi, 2\pi) \to \mathbb{R}^2$ be defined as $\alpha(t) = (\cos t, \sin t)$. Find the length of α. (2 marks)

2. If $f : U \to \mathbb{R}$ where $U \subseteq \mathbb{R}^{n+1}$ be an open subset, and $g : \mathbb{R} \to \mathbb{R}$ are two smooth functions prove that $d(h \circ f) = (h' \circ f)df$. (2 marks)

3. Define exact 1-form and prove that the integral over of an exact 1-form over a compact connected oriented plane curve is always zero. (3 marks)

Part B:

4. Define: principal curvature, principal curvature directions, first fundamental form, second fundamental form, negative definite and positive semi definite. (3 marks)
5. Justify: The second fundamental form of an oriented n- surface at p is positive definite if and only all the principal curvature of S at p are positive. (3 marks)
6. Find the Gaussian curvature of a cylinder over a plane curve. (2 marks)

Part B:

4. Define: principal curvature, principal curvature directions, first fundamental form, second fundamental form, negative definite and positive semi definite. (3 marks)
5. Justify: The second fundamental form of an oriented n- surface at p is positive definite if and only all the principal curvature of S at p are positive. (3 marks)
6. Find the Gaussian curvature of a cylinder over a plane curve. (2 marks)