Ordinary Differential Equations 2023 - Minor 3

First question carries 2 marks, second carries 4 and third and forth having 5 marks each

1. a. Write the standard forms of homogeneous and non homogeneous linear systems

   b. State a theorem which guarantees the existence and uniqueness of the n-th order equation

$$y^{(n)} = f(x, y, y', ...y^{(n-1)}).$$

2. If $W(t)$ is the Wronskian of two independent solutions of the following system

$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y. \end{cases}$$

   prove that

$$\frac{dW}{dt} = [a_1(t) + b_2(t)]W.$$

3. a. Show that

$$\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases} \text{and} \begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$$

   are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y. \end{cases}$$

   b. Show that the given solutions above are linearly independent on every closed interval, and write general solution of this system.

   c. Find the particular solution

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

   of this system for which $x(x) = 5$ and $y(0) = 1$.

4. Discuss the solution of

$$\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y. \end{cases}$$

   when the auxiliary equation has distinct complex roots and find its Wronskian.