Ordinary Differential Equations 2023 - Minor 2

PART A: First and second question carries 3 marks and 2 marks respectively.

1. Define the following
   (a) The n-th Legendre polynomial.
   (b) The orthogonal functions in an interval.
   (c) State Bessel expansion theorem.
2. What is the value of $1/m!$ when m is a negative integer. Explain.

PART B: Each question carries a maximum of 5 marks.

3. Minimize the integral with the right choice of p:

$$\int_{-1}^{1} (f(x) - p(x))^2 dx$$

   where $f(x)$ is function such that $f : [-1, 1] \to \mathbb{R}$ and $p(x)$ be a polynomial of degree 2. (Describe about the suitable p which minimizes the integral.)

4. If $p(x)$ is a polynomial of degree $n \geq 1$ such that

$$\int_{-1}^{1} x^k p(x) dx = 0 \quad \text{for } k = 0, 1, 2, .., n-1,$$

   show that $p(x) = cP_n(x)$ for some constant c.

PART C: Each question carries a maximum of 5 marks.

5. Show that J_m and J_{-m} are dependant solutions of Bessel's equation when m is a positive integer .
6. Prove that $\Gamma(1/2) = \sqrt{\pi}$.

Bonus Question: 5 Marks

7 Prove that $\lim_{x\to\infty} xe^{-x} = 0$.