Ordinary Differential Equations 2023 - Minor 3

First question carries 2 marks, second carries 4 and third and forth having 5 marks each

1. a. Explain the linear independence of solutions of

$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y. \end{cases}$$

   b. State a theorem which guarantees the existence and uniqueness of

$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t) \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t). \end{cases}$$

2. Show that the condition $a_2 b_1 > 0$ is sufficient, but not necessary, for the system

$$\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y. \end{cases}$$

   to have a real valued linearly in dependant solution of the form

$$\begin{cases} x = Ae^{mt} \\ y = Be^{mt} \end{cases}$$

3. Replace each of the following differential equations by an equivalent system of first order equations

   (a) $y'' - x^2y - xy = 0$

   (b) $y''' = y'' - x^2(y')^2$

4. Discuss the solution of

$$\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y. \end{cases}$$

   when the auxiliary equation has distinct complex roots and find its Wronskian.