First Order Partial Differential Equations
Method of Characteristics

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1. Find a smooth function $a : \mathbb{R}^2 \to \mathbb{R}$ such that, for the equation of the form

$$a(x, y)\, u_x + u_y = 0,$$

   there does not exist any solution in the entire $\mathbb{R}^2$ for any nonconstant initial value prescribed on $\{y = 0\}$.

2. Consider the PDE $xu_x + yu_y + zu_z = 3u$ in $\mathbb{R}^3$.
   (a) Solve the PDE with initial condition $u(x, y, 1) = x^2 + y^2$.
   (b) Is it possible to find unique solution if the initial condition is prescribed on the surface $z = 1 + x^2 + y^2$?

3. Consider the following IVPs:
   A. $u = u_x^2 - 3u_y^2,\ u(x, 0) = x^2,\ x > 0.$
   B. $u = u_x u_y,\ u(x, 0) = x^2,\ x > 0.$
   (a) Discuss the existence and uniqueness of both IVPs.
   (b) Solve any one the above.

4. Consider the PDE $xu_x + yu_y = 2u$ on $\mathbb{R}^2$. Discuss the existence and uniqueness of the solution (both global and local) to the following initial conditions. If unique solution does not exist, find an alternative solution on $\mathbb{R}^2$.
   (a) $u = 1$ on the hyperbola $xy = 1, x > 0$.
   (b) $u = 1$ on the line $y = 1$.
   (c) $u = 1$ on the circle $x^2 + y^2 = 1$.
   (d) $u(x, e^x) = xe^x$, for all $x \in \mathbb{R}$.

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