Partial Differential Equations 2024 - Assignment and Seminar Exam(Combined)
Each question carries 6 marks.

1. Estimate: For the fundamental solution Φ of the Laplacian and $n \geq 2$
$$\int_{\partial B(0,\epsilon)} |\Phi(y)| dy \leq C(\epsilon).$$

2. If $g \in C_c^\infty(\mathbb{R})$ and f is locally integrable, then $f * g$ exists and is infinitely differentiable on $\mathbb{R}$.

3. State Gauss-Green theorem. Let U be an open bounded subset of $\mathbb{R}^n$. Suppose that $u \in C^1(\bar{U})$, then
$$\int_U u\Delta v - v\Delta u dx = \int_{\partial U} u\frac{\partial v}{\partial \nu} - v\frac{\partial u}{\partial \nu} \quad dS$$

4. Define standard mollifier. Let U be an open subset of $\mathbb{R}$ and $f : U \to \mathbb{R}$ is locally integrable, define its mollification f^ϵ and prove that $f^\epsilon \to f$ a.e. as $\epsilon \to 0$.

Partial Differential Equations 2024 - Minor 2
Each question carries 4 marks.

1. Evaluate: For $x = (2, 0) \in \mathbb{R}^2$
$$\int_{B(x,\sqrt{2})} \ln\left(x^2 + y^2\right) \quad dxdy.$$

2. State true or false with justification: Let $u : \mathbb{R}^n \to \mathbb{R}$ be a continuous bounded function which satisfies mean value property, then u is a constant.

3. For the case $n = 2$, write the Laplace operator Δ in polar coordinates.

4. Let Ω be a open bounded subset of $\mathbb{R}^n$, $u : \Omega \to \mathbb{R}$ be a harmonic function, $A = \{x \in \Omega : u(x) = M\}$ where $M = \sup_{x\in\Omega} u(x)$. Then which of the following is/are correct:

   a. A is open in Ω.
   b. A is closed in Ω.
   c. A is empty.
   d. A is empty only if u is non-constant.