Real Analysis - Minor 1

Total Marks: 30 Time: 1 hour

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1. Prove that $\sup(A+B) = \sup A + \sup B$ (5)
2. Let $z > 0$ and $x < y$. Prove that there exist a rational number r such that $x < rz < y$. (5)
3. Prove that(using definition) (5)
$$\lim_{n\to\infty} \frac{2n}{3n^2+5} = 0$$
4. Let (x_n) be a bounded sequence and let $s := \sup\{x_n : n \in \mathbb{N}\}$. Show that if $s \notin \{x_n : n \in \mathbb{N}\}$, then there is a subsequence of (x_n) that converges to s. (5)
5. Define (5)
$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$
Prove that f is not continuous at any point of $\mathbb{R}$.
6. Prove that $\frac{1}{x}$ is not uniformly continuous on $(0,1)$. (5)