Kannur University

Department of Mathematical Sciences

Real Analysis - Minor 4

MSc 2025 Aug - Dec

Maximum Marks: To Be Determined By The Curve **Time: 60 Minutes**
Date: November 8, 2025

Instructions:

1. Attempt **all** questions.
2. Provide clear justifications, proofs, or counterexamples for every response.
3. State any theorems used clearly.

"You'll have bad times, but it'll always wake you up to the good stuff you weren't paying attention to."
— Good Will Hunting

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1. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be sequences of real numbers where a_n's are nonzero. Assume $\{a_n\}$ is convergent with $\lim a_n = L \neq 0$, and the product sequence $\{a_n b_n\}$ is also convergent. **Prove or disprove** the following statement: The sequence $\{b_n\}$ must be convergent.

2. Let the sequence $\{a_n\}_{n=1}^{\infty}$ be defined by $a_n = \frac{n}{n+1}$. Using the **formal definition of a Cauchy sequence**, prove that $\{a_n\}$ is a Cauchy sequence.

3. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers with the property: **Every subsequence of $\{a_n\}$ has a further subsequence that converges to zero. Prove** that the sequence $\{a_n\}$ itself converges to zero.

4. Let $\{x_n\}_{n=1}^{\infty}$ be an **unbounded sequence** of real numbers. **Prove or disprove** the following statement: There exists a subsequence $\{x_{n_k}\}_{k=1}^{\infty}$ such that the sequence of reciprocals $\left\{\frac{1}{x_{n_k}}\right\}_{k=1}^{\infty}$ converges to zero.

5. Consider the infinite series $\sum_{n=1}^{\infty} a_n$, where $a_n = \cos(n)$. **Prove or disprove** the following statement: The series $\sum_{n=1}^{\infty} \cos(n)$ diverges.

6. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be two sequences. Define $\{z_n\}$ by $z_{2n-1} = x_n$ and $z_{2n} = y_n$ (i.e., $\{x_1, y_1, x_2, y_2, \dots\}$). **Prove** that $\{z_n\}$ is convergent **if and only if** both $\{x_n\}$ and $\{y_n\}$ are convergent and $\lim x_n = \lim y_n$.

7. (a) **Prove** that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges.

   (b) Does the series $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$ converge or diverge? Justify your answer.