Kannur University
Department of Mathematical Sciences
Real Analysis - Minor 5
MSc 2025 Aug - Dec

Maximum Marks: To Be Determined By The Curve **Time: 60 Minutes**
Date: November 11, 2025

Instructions: Answer ALL Questions.

Q.1. Discuss the convergence or divergence of the following series, providing a clear proof for each.

(a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2}\right)$

Hint:

– Recall the Taylor expansion for $\ln(1+x)$: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$
– For question (a), think about the sequence of partial sums (S_k).
– For question (b), use the Mean Value Theorem for the natural logarithm on $[1, 1+x]$ and try to apply the Comparison Test.

Q.2. If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers, which of the following is/are true?

- $\limsup(x_n + y_n) \leq \limsup x_n + \limsup y_n$
- $\limsup(x_n + y_n) \geq \limsup x_n + \limsup y_n$
- $\liminf(x_n + y_n) \leq \liminf x_n + \liminf y_n$
- $\liminf(x_n + y_n) \geq \liminf x_n + \liminf y_n$

Hint: Definitions

– $\limsup a_n = \lim_{k\to\infty} (\sup\{a_n : n \geq k\})$
– $\liminf a_n = \lim_{k\to\infty} (\inf\{a_n : n \geq k\})$

(Also recall the relationship between $\liminf a_n$ and $-\limsup(-a_n)$.)

Q.3. Let $p(x)$ be a polynomial in the real variable x of degree 5. Then $\lim_{n\to\infty} \frac{p(n)}{2^n}$ is:

1. 5
2. 1
3. 0
4. ∞

Hint:

- Binomial Expansion : $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^n b^{n-k}$ (n is positive integer) for the expansion of 2^n

---

Q.4. The limit

$$\lim_{x\to 0} \frac{1}{x} \int_{x}^{2x} e^{-t^2} dt$$

1. does not exist.
2. is infinite.
3. exists and equals 1.
4. exists and equals 0.

Hint:

- The Taylor expansion of e^u around $u = 0$ is $e^u = 1 + u + \frac{u^2}{2!} + \ldots$.
