Kannur University

Department of Mathematical Sciences

Real Analysis - Minor 6

MSc 2025 Aug - Dec

Maximum Marks: To Be Determined By The Curve **Time: 60 Minutes**
Date: November 21, 2025

Instructions

1. **Answer all questions.**
2. **Approach:** Think about examples to get an idea about the result, whether it is true or not, or how it works. Take your time, think deeply, and conclude in the end. Use pen and paper and get your hands dirty.).

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1. (a) Show that if the series $(\sum x_n)$ converges **absolutely** and the sequence (y_n) is **bounded**, then the series $(\sum x_n y_n)$ **converges**.

   (b) Find a **counterexample** that demonstrates that (a) does not always hold if the convergence of $(\sum x_n)$ is **conditional**.

2. **True or false** (proof or counterexample): If $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_n^2$.

3. **True or false**, prove or find a counterexample. If $\{x_n\}$ is a sequence such that $\{x_n^2\}$ **converges**, then $\{x_n\}$ **converges**.

4. Show that if the series $(\sum a_k^2)$ **converges**, then the series $(\sum a_k^3)$ will **converge** as well.

5. Let $f : [a, b] \to \mathbb{R}$ be a **continuous function**. Then there exists c in (a, b) such that

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c).$$

   Hint: Consider using the **Mean Value Theorem** and the **Fundamental Theorem of Calculus**.
   This is the Mean Value Theorem for Integrals. You will encounter this theorem frequently if you continue studying mathematics.

6. **Riemann Integrability and Sums** Define f on $[0, 1]$ by

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ x^3 & \text{if } x \text{ is irrational} \end{cases}$$

   Select the correct option:

(1) f is not Riemann integrable on $[0,1]$.

(2) f is Riemann integrable and $\int_0^1 f(x)\,dx = \frac{1}{4}$.

(3) f is Riemann integrable and $\int_0^1 f(x)\,dx = \frac{1}{3}$.

(4) $\frac{1}{4} = \mathbf{L(f)} < \mathbf{U(f)} = \frac{1}{3}$.

Hint: The **upper Darboux integral $\mathbf{U(f)}$** of f is:

$$\mathbf{U(f)} = \inf\{U(f,P) : P \text{ is a partition of } [a,b]\}.$$

The **lower Darboux integral $\mathbf{L(f)}$** of f is:

$$\mathbf{L(f)} = \sup\{L(f,P) : P \text{ is a partition of } [a,b]\}.$$

(Where $U(f,P)$ and $L(f,P)$ are the upper and lower Darboux sums for a partition P.)

"How many times do I have to teach you: just because something works doesn't mean it can't be improved." -Black Panther