Set Theory and Functions: Assignment

FYIMP 2025 Batch - First Semester

Submission Deadline: 23rd October 2025, 3:30 PM

Instructions: Answer all questions. Show all necessary working for computational problems.

I. Set Basics and Operations (18 Questions)

Part A: Definitions and True/False

1. **Define** the power set of a set A. (Short Answer)
2. If a set A has n elements, what is the cardinality of its power set $P(A)$? (Fill in the blank)
3. **True or False:** For any non-empty sets A and B, $A \cap B \subseteq A \cup B$.
4. **True or False:** If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
5. **True or False:** The set $\{\emptyset\}$ is the same as the empty set $\emptyset$.
6. List all the subsets of the set $A = \{a, b\}$.

Part B: Computational Practice

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the universal set.
Let $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 4, 5, 6\}$, and $C = \{6, 7, 8, 9, 10\}$.

7. Find the set $A \cap B$.
8. Find the set $B \cup C$.
9. Find the complement of A, A^c.
10. Find the set difference $A \setminus B$.
11. Find the set $(A \cup B)^c$.
12. Find the set $A \cap (B \setminus C)$.
13. Calculate the cardinality: $|A \cap C|$.

14. Calculate the cardinality: $|P(B \cap C)|$.

15. Draw a Venn Diagram to illustrate the set operation $(A \cap B)^c$.

16. Given sets $X = \{x \mid x \text{ is an even integer}\}$ and $Y = \{y \mid y \text{ is a prime number}\}$. Describe the set $X \cap Y$.

17. If a set S has 4 elements, how many non-empty proper subsets does S have?

18. Shade the region corresponding to $A \setminus (B \cap C)$ in a three-set Venn diagram.

II. Cartesian Products and Relations (7 Questions)

19. If $A = \{a, b\}$ and $B = \{1, 2, 3\}$, list all the elements of the **Cartesian Product** $A \times B$.

20. What is the cardinality of $B \times A$?

21. **True or False:** For any two sets A and B, $A \times B = B \times A$.

22. Let $A = \{1, 2, 3\}$. A relation R is defined on A as $R = \{(x, y) \mid x \text{ divides } y\}$. List the ordered pairs belonging to R.

23. For the relation $R = \{(1, 5), (2, 6), (3, 7)\}$, state the **Domain** and the **Range**.

24. How many distinct relations can be defined from a set A with $|A| = 2$ to a set B with $|B| = 3$?

25. Given $S = \{2, 4, 6, 8\}$, define a relation R on S by xRy if $x + y \leq 10$. List the ordered pairs in R.

III. Functions (20 Questions)

Part A: Function Definitions and Types

26. **Define** a function from set A to set B.

27. **Define** a **bijective** function.

28. Which of the following sets of ordered pairs represents a function from $\{1, 2, 3\}$ to $\{a, b, c\}$? (Select all that apply)

    (a) $\{(1, a), (2, b), (1, c)\}$
    (b) $\{(1, a), (2, b), (3, a)\}$
    (c) $\{(1, a), (2, a)\}$

29. Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = x+1$. Is this function **one-to-one** (injective)? Justify your answer briefly.

30. Consider the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2$. Is this function **onto** (surjective)? Justify your answer briefly.

31. Give an example of a function $f : \{1, 2\} \to \{a, b, c\}$ that is **not surjective**.

32. Give an example of a function $g : \{1, 2, 3\} \to \{a, b\}$ that is **not injective**.

33. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 2x$. Is f a **bijection**? Why or why not?

Part B: Inverse and Composition

For questions 34-36, let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 4x - 5$.

34. Find the **inverse function**, $f^{-1}(x)$.

35. Compute $f(f^{-1}(7))$.

36. Compute $f^{-1}(-1)$.

For questions 37-40, let $f(x) = x + 2$ and $g(x) = 3x$.

37. Find the **composition** $(f \circ g)(x)$.

38. Find the **composition** $(g \circ f)(x)$.

39. Compute $(f \circ g)(1)$.

40. Is $f \circ g = g \circ f$? (**True/False**)

For questions 41-43, consider $h(x) = \frac{1}{x+1}$ and $k(x) = x^2$.

41. Find the domain of the function $h(x)$.

42. Compute $(h \circ k)(x)$.

43. Compute $(k \circ h)(0)$.

Part C: Simple Proofs

44. **Proof Skill:** Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 5x+2$ is **injective** (one-to-one).

45. **Proof Skill:** Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$ is **surjective** (onto).

— End of Assignment (Total: 45 Questions) —