Basic Measure Theory Problem Sheet

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1. Let $f : [0, 1] \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q} \\ 1 & \text{otherwise} \end{cases}$$

Then, which of the following are correct?

(A) f is not Riemann integrable

(B) f is Riemann integrable

(C) Limit of upper sums of f is one

(D) Limit of lower sums of f is zero

2. Let $A_n, n = 1, 2, 3, \ldots$ be a sequence of subsets of a set X. Define $\limsup A_n = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$. Then, which of the following are always true?

(A) $\limsup A_n = \{x \in X : x \in A_k \text{ for infinitely many } k\}$

(B) $\limsup A_n = \{x \in X : x \in A_k \text{ only for finitely many } k\}$

(C) $\limsup A_n = \{x \in X : x \in A_k \forall k\}$

(D) $\limsup A_n = \{x \in X : x \in A_k \text{ for all but finitely many } k\}$

3. Let $A_n, n = 1, 2, 3, \ldots$ be a sequence of subsets of a set X. Define $\liminf A_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$. Then, which of the following are always true?

(A) $\liminf A_n = \{x \in X : x \in A_k \text{ for infinitely many } k\}$

(B) $\liminf A_n = \{x \in X : x \in A_k \text{ only for finitely many } k\}$

(C) $\liminf A_n = \{x \in X : x \in A_k \forall k\}$

(D) $\liminf A_n = \{x \in X : x \in A_k \text{ for all but finitely many } k\}$

4. Let X be a set and let A, B be proper distinct subsets of X. Consider the sequence of sets

$$A_n = \begin{cases} A & \text{if } n \text{ is even} \\ B & \text{if } n \text{ is odd} \end{cases}$$

Which of the following are correct?

(A) $\limsup A_n = A \cup B$

(B) $\limsup A_n = A \cap B$

(C) $\liminf A_n = A \cup B$

(D) $\liminf A_n = A \cap B$

5. Which of the following are true?

(A) $\mathbb{Q} \in \mathcal{B}(\mathbb{R})$

(B) $\{x : 0 \leq x \leq 1 \text{ and } x \text{ is irrational}\} \in \mathcal{B}(\mathbb{R})$

(C) For $a < b$, $[a, b) \in \mathcal{B}(\mathbb{R})$

(D) $[0, 1] \times [0, 1] \in \mathcal{B}(\mathbb{R}^2)$

6. Which of the following are true?

(A) $\{(x, 0) : x \in \mathbb{R}\} \in \mathcal{B}(\mathbb{R}^2)$

(B) $\{(x, x) : x \in \mathbb{R}\} \in \mathcal{B}(\mathbb{R}^2)$

(C) $\{(x, y) : y = x^2 + x, x \in \mathbb{R}\} \in \mathcal{B}(\mathbb{R}^2)$

(D) $A \times \{0\} \in \mathcal{B}(\mathbb{R}^2)$ for any finite set $A \subset \mathbb{R}$.

7. Which of the following are true?

(A) $\mathcal{B}(\mathbb{R}^n)$ is generated by open balls of rational radii

(B) $\mathcal{B}(\mathbb{R}^n)$ is generated by closed balls of rational radii

(C) $\mathcal{B}(\mathbb{R}^n)$ is generated by singletons

(D) $\mathcal{B}(\mathbb{R})$ is generated by $\{[a, b) : a, b \in \mathbb{Q}\}$

8. Which of the following are $\sigma-$algebras?

(A) $\{A \subset \mathbb{R} : A \text{ is countable or } A^c \text{ is countable}\}$

(B) $\{A \subset \mathbb{R} : A \text{ is finite or } A^c \text{ is finite}\}$

(C) $\{A \subset \mathbb{R} : A \text{ is a closed interval}\}$

(D) $\{A : A \subset [0, 1] \text{ and } A \in \mathcal{B}(\mathbb{R})\}$

9. Which of the following definitions give a measure on $\mathcal{B}(\mathbb{R})$?

(A) $\mu(A) =$ number of rationals in A

(B) $\mu(A) = 1$ if $1 \in A$ and 0 otherwise

(C) $\mu(A) =$ number of rationals in $A \cap [0, 1]$

(D) $\mu(A) =$ number of rationals in A^c

10. Let $n > 1$ and let $X = \{1, 2, \ldots, n\}$. Let $\mathcal{F}$ be the powerset of X and μ the counting measure on $\mathcal{F}$. If $f : X \to \mathbb{R}$, then which of the following are true?

(A) $\int_X f d\mu = \sum_{k \in X} f(k)$

(B) $\int_X f d\mu = f(1)$

(C) $\int_X f d\mu = 0$

(D) $\int_X f d\mu = 2f(1)$

11. Let $X = \mathbb{R}, \mathcal{F} = \{A \subset \mathbb{R} \mid A \text{ is countable or } A^c \text{ is countable}\}$. Let $\mu(A) = 1$ if A^c is countable and 0 if A is countable. Let $f : (X, \mathcal{F}, \mu) \to \mathbb{R}$ be measurable. Which of the following are always true?

(A) f is constant ae.(μ)

(B) f is a non-constant continuous function

(C) f is a non constant polynomial

(D) $f(x) = 0 \forall x \in \mathbb{R}$

12. Let $(X, \mathcal{F}, \mu)$ be a measure space and let E be a proper subset of $X, E \in \mathcal{F}$ and $0 < \mu(E) < \mu(X)$. Let $f_n = \chi_E$ if n is odd and $1 - \chi_E$ if n is even. Which of the following are true?

(A) $\int_X \liminf f_n d\mu < \liminf \int_X f_n d\mu$

(B) $\int_X \liminf f_n d\mu = \liminf \int_X f_n d\mu$

(C) $\int_X \limsup f_n d\mu < \limsup \int_X f_n d\mu$

(D) $\int_X \limsup f_n d\mu = \limsup \int_X f_n d\mu$

13. Consider the space $\mathbb{N}$ with power set sigma algebra and counting measure μ. Let $f : \mathbb{N} \to \mathbb{R}$ be measurable and zero ae(μ). Which of the following are always true?

(A) $f(n) = 0 \forall n \in \mathbb{N}$

(B) $f(1) \neq 0, f(n) = 0 \forall n > 1$

(C) $f(n) = 0$ except for finitely many $n \in \mathbb{N}$

(D) $f(n) = 0$ only when n is a prime number

14. Let X be a non empty set and $A \subset X$ be a proper subset. Consider the sigma algebra $\mathcal{F} = \{\emptyset, X, A, A^c\}$. Let $f : (X, \mathcal{F}) \to \mathbb{R}$ be measurable. Which of the following are always true?

(A) $f = \alpha\chi_A + \beta\chi_{A^c}$ for some $\alpha, \beta \in \mathbb{R}$

(B) $f = \alpha\chi_A$ for some $\alpha \in \mathbb{R}$

(C) $f = \beta\chi_{A^c}$ for some $\beta \in \mathbb{R}$

(D) $f \equiv 0$

15. Let $(X, \mathcal{F}, \mu)$ be a measure space. Let $A_n \in \mathcal{F}$ such that $A_1 \subset A_2 \subset \ldots$ and $\cup_{n=1}^{\infty} A_n = X$. Let $f : (X, \mathcal{F}, \mu) \to \mathbb{R}$ be a measurable function and $f(x) \geq 0$ ae(μ). Which of the following are always true?

(A) $f\chi_{A_n} \uparrow f$ ae

(B) $\int_{A_n} f d\mu \uparrow \int_X f d\mu$

(C) $\int_{A_n} f d\mu \downarrow \int_X f d\mu$

(D) $f\chi_{A_n} \downarrow f$

16. Let $(X, \mathcal{F}, \mu)$ be a probability measure space ($\mu(X) = 1$). Let $\{A_n\}$ be a sequence in $\mathcal{F}$. Which of the following are true?

(A) $\mu(\limsup A_n) \geq \limsup \mu(A_n)$

(B) $\mu(\limsup A_n) \leq \limsup \mu(A_n)$

(C) $\mu(\liminf A_n) \geq \liminf \mu(A_n)$

(D) $\mu(\liminf A_n) \leq \liminf \mu(A_n)$

17. Let $(X, \mathcal{F}, \mu)$ be a measure space. Let $f_n : X \to \mathbb{R}$ be measurable, $A = \{x \in X \mid \lim f_n(x) \text{ exists}\}$. Then,

(A) $A \in \mathcal{F}$

(B) $A = \emptyset$

(C) $A = X$

(D) $A^c \in \mathcal{F}$

18. Let $(X, \mathcal{F}, \mu)$ be a measure space and $A_n \in \mathcal{F}$. Suppose $\mu(X) = 1, \sum \mu(A_n) < \infty$. Then which of the following are true?

(A) $\mu(\limsup A_n) = 0$

(B) $\mu(\liminf A_n) = 0$

(C) $\mu(\limsup A_n) = 1$

(D) $\mu(\liminf A_n) = 1$

19. $(X, \mathcal{F}, \mu)$ be a probability measure space. Suppose $f_n : X \to \mathbb{R}$ are measurable and $|f_n| \leq 1$ ae$(\mu)\forall n$. Suppose $f_n \to 1$ ae(μ). Then,

(A) $\int_X f_n d\mu \to 1$

(B) $\int_X f_n d\mu \to 0$

(C) $\int_X f_n d\mu \to \infty$

(D) $\int_X f_n d\mu$ does not converge

20. Let $(X, \mathcal{F}, \mu)$ be a measure space and $A_n \in \mathcal{F}$ such that $\mu(A_n) = 0 \forall n$. Which of the following are true?

(A) $\mu(\cup_{n=1}^{\infty} A_n) = 0$

(B) $\mu(\cup_{n=1}^{\infty} A_n) = 1$

(C) $\mu(\cup_{n=1}^{\infty} A_n) = \infty$

(D) $\mu(\cup_{n=1}^{\infty} A_n) > 0$

21. Let F be a countable subset of $\mathbb{R}^n$ then $m_*(F)$ equals:

(A) 0

(B) ∞

(C) any positive number

22. Which of the following are correct?

(A) Any countable set is measurable

(B) Any open set is measurable

(C) The set $\{x \in \mathbb{R} : |e^x \sin x| > 1\}$ is measurable

23. Which of the following sets are Borel sets?

(A) The subset of $[0, 1]$ whose decimal expansions starts with 2

(B) Subsets of $\mathbb{R}$ whose complements are countable

24. The outer measure of the set $\{0\} \times [-1, 1] \subset \mathbb{R}^2$ is:

(A) 0

(B) 1

(C) 2

25. Let $E \subset \mathbb{R}^n$ be an unbounded set.

(A) Outer measure of E is infinity

(B) Outer measure of E is positive, but need not be infinity always

(C) There are unbounded sets whose outer measure is zero

26. Let $E \subset \mathbb{R}^n$ be such that $m_*(E) = 0$. Let O_n be the open set $O_n = \{y \in \mathbb{R}^n : d(y, E) < \frac{1}{n}\}$.

(A) $m_*(O_n) = \infty$ always

(B) $m_*(O_n)$ is finite always

(C) $m_*(O_n)$ is positive always

27. Let $E \subset \mathbb{R}^n$ be such that $m_*(E) = 0$ and O_n as defined above. Then the statement $m_*(O_n) \to 0$ is:

(A) True always

(B) True if E is closed

(C) True if E is bounded

28. Let $E \subset \mathbb{R}^n$ and $x \in \mathbb{R}^n$. Define $E + x = \{y + x : y \in E\}$. Suppose $m_*(E) = 0$. Which are correct?

(A) $m_*(E + x) = 0$.

(B) $E + x$ is measurable

(C) $E + x$ need not be measurable

29. Let $A_n = \{n\} \times [-n, n] \subset \mathbb{R}^2$. If $A = \cup_{n=1}^{\infty} A_n$, then:

(A) $m_*(A) = 0$

(B) $m_*(A) = \infty$

(C) $0 < m_*(A) < \infty$

30. Let $A = [0, 1]$. Which of the following are correct?

(A) $m_*(A) = \inf\{m_*(O) : A \subset O, O \text{ open}\}$

(B) $m_*(A) = \sup\{m_*(K) : K \subset A, K \text{ compact}\}$

31. Let A be a subset of $[0, 1]$ and m denote Lebesgue measure. Then which of the following are true?

(A) If A is closed then $m(A) > 0$

(B) If A is open then $m(A) = m(\bar{A})$

(C) If $m(int(A)) = m(\bar{A})$ then A is measurable

(D) If $m(int(A)) = m(\bar{A})$ then A need not be measurable

32. Define an equivalence relation in $[1, 2]$ by $x \sim y$ if $x - y \in \mathbb{Q}$. Let N contain one element from each class.

(A) N is uncountable

(B) $[1, 2] \setminus N$ is uncountable

(C) $m_*(N) = 0$

(D) $E \subset N$ measurable implies $m_*(E) = 0$

33. Let $f : \mathbb{R} \to \mathbb{R}$. Which are necessarily true?

(A) If f is measurable, then $\phi \circ f$ is measurable for any continuous ϕ

(B) If f^2 is measurable, then f is measurable

(C) If f is differentiable, then f' is measurable

(D) If g is measurable and $f = g$ ae, then f is measurable

34. Let $\{f_n\}$ be real valued functions on $[0, 1]$ converging pointwise to a continuous function f.

(A) $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$

(B) $0 \le f_n \le f \implies \lim_{n\to\infty} \int f_n = \int f$

(C) $|f_n(x)| \le 1/\sqrt{x} \implies \lim_{n\to\infty} \int f_n = \int f$

(D) $|f_n(x)| \le 1 \implies \lim_{n\to\infty} \int_K f_n = \int_K f$ for measurable K

35. Let $\{f_n\}, \{g_n\}, f, g \in L^1(\mathbb{R}^n)$ with $f_n \to f, g_n \to g$ ae.

(A) $\int(f_n + g_n) \to \int(f + g)$

(B) $|f_n| \le |f|, |g_n| \le |g| \implies \int(f_n + g_n) \to \int(f + g)$

(C) $|f_n| \le |g| \implies \int f_n \to \int f$

(D) $|f_n| \le |g_n|$ and $\int f_n \to \int f \implies \int g_n \to \int g$

36. Let $f_n(x) = e^{-nx^2}$ on $[1, \infty)$. Which are true?

(A) $\int_1^\infty f_n(x)dx \to 0$

(B) $\sup_n \|f_n\|_1 < \infty$

(C) f_n converges in $L^1[1, \infty)$

(D) f_n does not converge in L^p for any $1 \le p \le \infty$

37. Let $m(E_n) \to 0$ and $f \ge 0$ be measurable.

(A) $\int_{E_n} f \to 0$

(B) If $E_{n+1} \subset E_n$ then $\int_{E_n} f \to 0$

(C) If f is bounded, then $\int_{E_n} f \to 0$

(D) If f is integrable and $E_{n+1} \subset E_n$, then $\int_{E_n} f \to 0$

38. Let $f, f_n : (X, \mathcal{F}, \mu) \to \mathbb{R}$ be measurable.

(A) $0 \leq f_n \to f$ uniformly $\implies \lim \int f_n = \int f$

(B) $\mu(X) < \infty, |f_n| \leq 1 \implies \lim \int g \circ f_n = \int g \circ f$ for continuous g

(C) $\mu(X) < \infty, |f_n| \leq 1, f_n \to f$ ae $\implies \lim \int f_n = \int f$

(D) $f_n \uparrow f \implies \lim \int f_n = \int f$

39. Let $\{f_n\}$ be measurable and converge uniformly to f on $\mathbb{R}$.

(A) $\lim \int_{-\infty}^{\infty} f_n = \int_{-\infty}^{\infty} f$

(B) $\lim \int_1^{\infty} f_n = \int_1^{\infty} f$

(C) $\lim \int_1^2 f_n = \int_1^2 f$

(D) $\lim \int_K f_n = \int_K f$ for compact K

40. Let $A \in \mathcal{L}(\mathbb{R}^n)$. Which are correct?

(A) $\delta A \in \mathcal{L}(\mathbb{R}^n)$ for all $\delta > 0$

(B) $A + x \in \mathcal{L}(\mathbb{R}^n)$ for all $x \in \mathbb{R}^n$

41. Let $f : \mathbb{R} \to \mathbb{R}$ s.t. $f^{-1}(\{a\})$ is a Lebesgue set $\forall a \in \mathbb{R}$.

(A) f is Lebesgue measurable always

(B) f is Borel measurable always

(C) $f : (\mathbb{R}, 2^{\mathbb{R}}) \to \mathbb{R}$ is measurable

42. Let $f_n : [0,1] \to \mathbb{R}$ be Riemann integrable and $f_n \to f$ pointwise.

(A) f is Riemann integrable

(B) f is Riemann integrable and $\lim \int f_n = \int f$

(C) f is Lebesgue measurable

43. Continuous $f_n \to$ continuous f pointwise on $\mathbb{R}$.

(A) $\lim \int_0^1 f_n = \int_0^1 f$

(B) $0 \leq f_n \leq f \implies \lim \int_a^b f_n = \int_a^b f$

(C) $|f_n| \leq e^{-x} \implies \lim \int_a^{\infty} f_n = \int_a^{\infty} f$

(D) $|f_n| \leq 1 \implies \lim \int_a^{\infty} f_n = \int_a^{\infty} f$

44. Let $f_n, g_n, f, g \in L^1(\mathbb{R}^k)$ with $f_n \to f, g_n \to g$ pointwise ae.

(A) $\int f_n \to \int f$ and $\int g_n \to \int g$

(B) $|f_n| \leq |g_n| \implies \int f_n \to \int f$

(C) $|f_n| \leq |g| \implies \int f_n \to \int f$

(D) $|f_n| \le |g_n|$ and $\int f_n \to \int f \implies \int g_n \to \int g$

45. $f_n(x) = e^{-nx^2}$ on $[1, \infty)$.

(A) $\int_1^\infty f_n \to 0$

(B) $\sup \|f_n\|_1 < \infty$

(C) $f_n \to 0$ in $L^1[1, \infty)$

(D) f_n doesn't converge in L^p for any p

46. Which are correct?

(A) χ_E is measurable iff E is a Lebesgue set

(B) χ_E is Borel measurable iff E is a Borel set

47. Integrability properties:

(A) f Riemann integrable, $g = f$ except at finite points $\implies$ g is R-integrable

(B) f continuous, $g = f$ except on countable set $\implies$ g is R-integrable

(C) f continuous, $g = f$ except on countable set $\implies$ g is L-integrable

48. Which are σ-finite?

(A) Counting measure on $([0, 1], 2^{[0,1]})$

(B) Counting measure on $([0, 1], \mathcal{B}[0, 1])$

(C) $\mu(A) = \#(A \cap \mathbb{Q})$ on Borel sets

(D) $\mu(A) = \#(A \cap \mathbb{Z})$ on Borel sets

49. Positive linear functionals on C_c or $C[0, 1]$?

(A) $T(f) = \sum f(1/n)$ on $C[0, 1]$

(B) $T(f) = \sum f(n)$ on $C_c(\mathbb{R})$

(C) $T(f) = \sum f(1/n)$ on $C_c(0, \infty)$

50. On $X = \mathbb{R} \setminus \{0\}$, which are positive linear functionals on $C_c(X)$?

(A) $T(f) = \sum f(1/k) + \sum f(-1/k)$

(B) $T(f) = \sum f(-k) + \sum f(k)$

(C) $T(f) = \sum f(k + 1/k)$

51. Lebesgue measure zero sets?

(A) Any countable subset of $\mathbb{R}$

(B) Cantor set

(C) Irrationals in Cantor set

(D) k-dim subspace of $\mathbb{R}^n$ $(k < n)$

52. Invertible linear map T on $\mathbb{R}^n$:

(A) E Lebesgue $\implies T(E)$ Lebesgue

(B) E Borel $\implies T(E)$ Borel

53. Invariance properties:

(A) Outer measure m^* is translation invariant

(B) Lebesgue measure m is translation invariant

(C) Borel measure μ with finite compact values is a multiple of Lebesgue

54. Borel measure μ where $\mu(A+n) = \mu(A) \forall n \in \mathbb{Z}$:

(A) μ is zero measure

(B) μ is constant multiple of Lebesgue

(C) Counting measure satisfies this

55. Linear $A : \mathbb{R}^n \to \mathbb{R}^n$:

(A) A singular $\implies A(E)$ is Lebesgue for all $E \subset \mathbb{R}^n$

(B) A invertible $\implies A(E)$ is Lebesgue for all $E \subset \mathbb{R}^n$

(C) A invertible $\implies A(E)$ is Lebesgue for all Lebesgue E

56. Positive measurable f on $\mathbb{R}^n$:

(A) $\int_{E+x_0} f = \int_E f$

(B) $\int f(x + x_0) = \int f$

(C) $\int f(Ax) = \int f$ for any linear A

57. Open/Closed sets in $[0, 1]$:

(A) Dense open $O \implies m(O) = 1$

(B) Open $O \implies m(O) > 0$

(C) Closed F with no interior $\implies m(F) = 0$

58. Measure zero in $\mathbb{R}^2$?

(A) $C \times \mathbb{R}$ (C is Cantor)

(B) $\mathbb{Q} \times \mathbb{R}$

(C) Countable union of lines through origin

59. Positive Lebesgue measure?

(A) Any unbounded set in $\mathbb{R}^2$

(B) Any unbounded closed set

(C) $(\mathbb{R} \setminus \mathbb{Q}) \times \mathbb{R}$

60. Cantor set:

(A) Closed

(B) Perfect
(C) Complement in $[0, 1]$ has measure 1
(D) Uncountable

61. L^p Space inclusions/products:

(A) $f, g \in L^1 \implies fg \in L^1$
(B) $f, g \in L^2 \implies fg \in L^2$
(C) $f \in L^1 \cap L^\infty \implies f \in L^2$

62. L^p Comparisons:

(A) $L^1[0, 1] \subset L^2[0, 1]$
(B) $L^1[0, \infty) \subset L^2[0, \infty)$
(C) $L^2[0, 1] \subset L^1[0, 1]$
(D) $L^2[0, \infty) \subset L^1[0, \infty)$

63. Let $f = \chi_{[0,1/2]}$:

(A) f is continuous ae
(B) f can be approximated by continuous functions in L^∞ norm
(C) $f = g$ ae for some continuous g

64. Radial function integrability:

(A) $\chi_{|x|\leq 1}|x|^a \in L^1(\mathbb{R}^n)$ iff $a > -n$
(B) $\chi_{|x|\leq 1}|x|^a \in L^1(\mathbb{R}^n)$ iff $a < -n$
(C) $\chi_{|x|\geq 1}|x|^a \in L^1(\mathbb{R}^n)$ iff $a > -n$
(D) $\chi_{|x|\geq 1}|x|^a \in L^1(\mathbb{R}^n)$ iff $a < -n$

65. More radial functions:

(A) $\chi_{|x|\leq 1}|x|^a(1 - |x|)^b \in L^1$ iff $a > -n, b > -1$
(B) $\chi_{|x|\leq 1}|x|^a(1 - |x|)^b \in L^1$ iff $a > -n, b > 0$
(C) $\chi_{|x|\geq 1}|x|^a(1 - |x|)^b \in L^1$ iff $a < -n, b > -1$

66. $f_n(x) = x^n$ on $[0, 1]$:

(A) $f_n \to 0$ uniformly
(B) $f_n \to 0$ in L^1
(C) $f_n \to 0$ in L^p for $1 \leq p < \infty$

67. $f_n(x) = x^{-n}$ on $[1, \infty)$:

(A) $f_n \to 0$ uniformly
(B) $f_n \to 0$ in L^1
(C) $f_n \to 0$ in L^p for $1 \leq p < \infty$

68. $f_n(x) = x^{-n}$ on $[2, \infty)$:

(A) $f_n \to 0$ uniformly

(B) $f_n \to 0$ in L^1

(C) $f_n \to 0$ in L^p for $1 \leq p \leq \infty$

69. L^p Norm properties:

(A) $p < r < s \implies L^p \cap L^s \subset L^r$

(B) $\mu(X) < \infty, r < p \implies L^p \subset L^r$

(C) $\mu(X) < \infty, f \in L^\infty \implies \|f\|_p \to \|f\|_\infty$

70. Positive measurable f, g with $fg \geq a > 0$:

(A) $\mu(X) = 1 \implies (\int f)(\int g) \geq a$

(B) $\mu(X) < 1 \implies (\int f)(\int g) \geq a$

71. Measurable $f : \mathbb{R} \to \mathbb{R}$:

(A) $f \in L^1 \implies f$ is bounded

(B) $f \in L^1$ and continuous $\implies f$ bounded

(C) $f \in L^1$ and continuous $\implies \lim_{|x| \to \infty} f(x) = 0$

(D) $f \in L^1$ and uniformly continuous $\implies f$ bounded

72. Product sigma algebras:

(A) $\mathcal{B}(\mathbb{R}) \times \mathcal{B}(\mathbb{R}) = \mathcal{B}(\mathbb{R}^2)$

(B) $\mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}) = \mathcal{L}(\mathbb{R}^2)$

73. Rectangles and sections:

(A) σ-algebra of rational rectangles is $\mathcal{B}(\mathbb{R}^n)$

(B) σ-algebra of rational rectangles is $\mathcal{L}(\mathbb{R}^n)$

(C) $E \in \mathcal{L}(\mathbb{R}^2) \implies E_x \in \mathcal{L}(\mathbb{R})$

74. Lebesgue measurable $f : \mathbb{R}^2 \to \mathbb{R}$:

(A) f_x is Lebesgue measurable $\forall x$

(B) f Borel $\implies f_x$ Borel

75. Product measure $m \times \mu$ (μ counts rationals in $[0, 1]$). Let D be diagonal:

(A) $(m \times \mu)(D) = 0$

(B) $(m \times \mu)(D) = \infty$

(C) $(m \times \mu)(D) = 1$

76. Product measure $\mu \times \mu$ (μ counts rationals). Let D be diagonal in $[0, 1]$:

(A) $(\mu \times \mu)(D) = 0$

(B) $(\mu \times \mu)(D) = \infty$
(C) $(\mu \times \mu)(D) = 1$

77. $f(x, y) = 1$ if $x = y$ on $[0, 1] \times [0, 1]$:

(A) $\int_X f(x, y)dm(x) = 0$
(B) $\int_Y f(x, y)d\mu(y) = 1$
(C) Iterated integrals are same
(D) μ not σ-finite $\implies$ iterated integrals not same

78. Subgraph $A(f) = \{(x, y) : 0 < y < f(x)\}$:

(A) $A(f) \in \mathcal{B}(\mathbb{R}^2)$
(B) $m(A(f)) = \int f(x)dx$

79. Graph $G(f) = \{(x, f(x))\}$:

(A) $G(f)$ is Borel
(B) $m(G(f)) = 0$
(C) $m(G(f)) = \infty$

80. Distribution function $F_f(t) = \mu\{f > t\}$:

(A) F_f is non-increasing
(B) $\int f d\mu = \int_0^\infty F_f(t)dt$

81. Complete measure spaces:

(A) $(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2), m)$
(B) $(\mathbb{R}^2, \mathcal{L} \times \mathcal{L}, m)$
(C) $(\mathbb{R}^2, \mathcal{L}(\mathbb{R}^2), m_2)$

82. $g(x) = \int_x^b \frac{f(t)}{t} dt$:

(A) g not integrable on $[0, b]$
(B) g integrable and $\int_0^b g = \int_0^b f$

83. $f(x) = |x|^a$ for $|x| \leq 1$:

(A) $\int f < \infty$ if $a > -n$
(B) $\int f < \infty$ if $a < -n$
(C) $\int f < \infty$ if $a > 0$

84. $f(x) = |x|^a$ for $|x| \geq 1$:

(A) $\int f < \infty$ if $a > -n$
(B) $\int f < \infty$ if $a < -n$
(C) $\int f < \infty$ if $a > 0$

85. $f(x) = (1 - |x|^2)^a$ for $|x| \leq 1$:

(A) $\int f < \infty$ if $a > -n$

(B) $\int f < \infty$ if $a > -1$

(C) $\int f < \infty$ if $a > 0$

86. $F(x) = \int_x^{x+1} g(y)dy$ where $g \in L^1$:

(A) $F \in L^1$

(B) F is bounded

(C) $\lim_{|x|\to\infty} F(x) = 0$

87. $F(x) = \sum_{k=-\infty}^{\infty} f(x+k)$ on $[0,1]$:

(A) $F \in L^1[0,1]$

(B) $F \in L^1 \implies f = 0$ ae

(C) F is finite ae

88. If $\int |f_k - f| \to 0$:

(A) $f_k \to f$ ae

(B) Subsequence $f_{k_j} \to f$ ae

(C) $f_k \to f$ in measure